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ORIGINAL ARTICLE

Experimental validation of physical-mathematical models of natural drying of laterite ore

Validación experimental de modelos físico-matemáticos del secado natural de mena laterítica

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Abstract

The process of natural drying of laterite ore has been the subject of modeling with the purpose of perfecting the technology used in its implementation. However, the models currently used for the energetic analysis of the process have not been experimentally validated. In order to solve this unsolved problem, the present work is developed in which, through the implementation of a multifactorial design of experiments, the experimental validation of the physical-mathematical models of the natural drying of laterite ore was developed. It was corroborated that the models describe the physical reality of the process with high accuracy (93.43 %), which can be improved through multidimensional modeling (2D and 3D) of the heat and mass transfer processes that characterize the object of study.

Keywords: design of experiments, experimental validation, mathematical models, natural drying, laterite ore

Resumen

El proceso de secado natural de la mena laterítica ha sido objeto de modelación con el propósito de perfeccionar la tecnología usada en su implementación. Sin embargo, los modelos actualmente utilizados para el análisis energético del proceso no han sido validados experimentalmente. Para dar solución a este problema se realiza el presente trabajo en el cual, mediante la implementación de un diseño de experimentos multifactorial, se desarrolló la validación experimental de los modelos físico-matemáticos del secado natural de la mena laterítica. Se corroboró que los modelos describen la realidad física del proceso con elevada precisión (93,43 %), la cual puede ser mejorada a través de la modelación multidimensional (2D y 3D) de los procesos de transferencia de calor y masa que caracterizan al objeto de estudio.

Palabras clave: diseño de experimentos, validación experimental, modelos matemáticos, secado natural, mena laterítica

1. INTRODUCTION

Solar drying, due to its diverse and sustainable applications, is widely studied internationally (Rao *et al*., 2025; Xu and Liu, 2025; Choubey *et al*., 2025; Nnamchi *et al*., 2025; Morya *et al*., 2025; Karadirek *et al*., 2025; Odoi-Yorke *et al*., 2025). In the lateritic ore (mineral earth) processed by the Cuban nickel industry, natural drying (wind and solar) has been the subject of several studies. Early work focused on establishing a technology for processing the material in the mining stockpiles of the Empresa Productora de Níquel y Cobalto Comandante Ernesto Che Guevara in Moa (Estenoz, 2001; Estenoz *et al*. 2004, 2006).

Subsequently, significant research (Estenoz *et al*., 2005, 2007a, 2007b; Estenoz, 2009; Fuentes and Estenoz-Mejia, 2020) dedicated to the practical implementation of the process at the aforementioned production entity was developed by a multidisciplinary group from the Nickel Research Development Center (CEDINIQ). This work designed a technology for the natural drying of lateritic ore, which outlines the formation, management, and operational control of mineral piles in mining stockpiles.

Although the designed technology has several advantages, it presents limitations (Retirado-Mediaceja, 2012; Retirado-Mediaceja *et al*., 2018), such as: it presupposes the construction of auxiliary facilities with high energy consumption and does not consider a rigorous evaluation of the heat and mass transfer processes affecting natural drying, nor the application of mathematical models tailored to the conditions under which the process occurs in the nickel industry. Furthermore, although this is a practice implemented worldwide with other materials (Javaherdeh *et al*., 2006; Montoya and Jiménez, 2006; Parra-Coronado *et al*., 2008; Salinas *et al*., 2008; Sandoval-Torres, 2009; Bergues-Ricardo *et al*., 2014; Pandey *et al*., 2024), it does not allow for predicting the moisture variation experienced by the material during the natural drying process and does not incorporate the characterization of the cross-sectional geometry of the piles.

The aforementioned inconsistencies in the technology were addressed in research developed at the University of Moa through mathematical modeling, simulation, and energy optimization of the process (Retirado-Mediaceja and Legrá, 2011; Retirado-Mediaceja *et al*., 2012a and 2012b). However, despite the proven effectiveness demonstrated by the established models (Retirado-Mediaceja *et al*., 2014, 2015, and 2016), no studies aimed at their experimental validation—based on implementing an experimental design adjusted to the specific characteristics of the process—have been reported.

The validation of mathematical models is of great importance as it determines the accuracy with which they correspond to the physical reality of the investigated process (Brito-Vallina *et al*., 2011). Validation can be performed by comparing them with data reported by other validated models, assessing the conclusions obtained when using the model in question, or comparing the results from the models with available experimental data on the subject of study (Legrá and Silva, 2011). The present work is dedicated to this latter aspect and has the following objective: to develop the experimental validation of the established physico-mathematical models for the energy analysis of the natural drying process of lateritic ore processed in the Cuban nickel industry.

The subsequent sections of the article present the materials and methods, which include the models and experimental data used in the validation. Finally, the obtained results are analyzed, future research lines are defined, and conclusions are drawn on the most relevant aspects addressed.

2. MATERIALS AND METHODS

2.1. Physico-Mathematical Models of Natural Solar Drying

The mathematical modeling of the natural drying process for lateritic ore allows for its theoretical study and, following corresponding experimental validation, enables computational simulations of the process using appropriate software systems (Retirado-Mediaceja *et al*., 2017).

The present study considered the modeling developed by Retirado-Mediaceja *et al*. (2012b) and incorporated the systematization of the calculation for the capture area, the transferred heat fluxes, and the distribution of temperature and moisture experienced by the material in the piles, $H(y,\tau)$ (Figure 1). The proposed model is obtained by solving the heat conduction and moisture equations using the method of separation of variables (Jiménez, 1999). This is the most widely used method and involves dividing the dependent variable into functions of a single independent variable to solve them separately (Polyanin and Manzhirov, 2022).

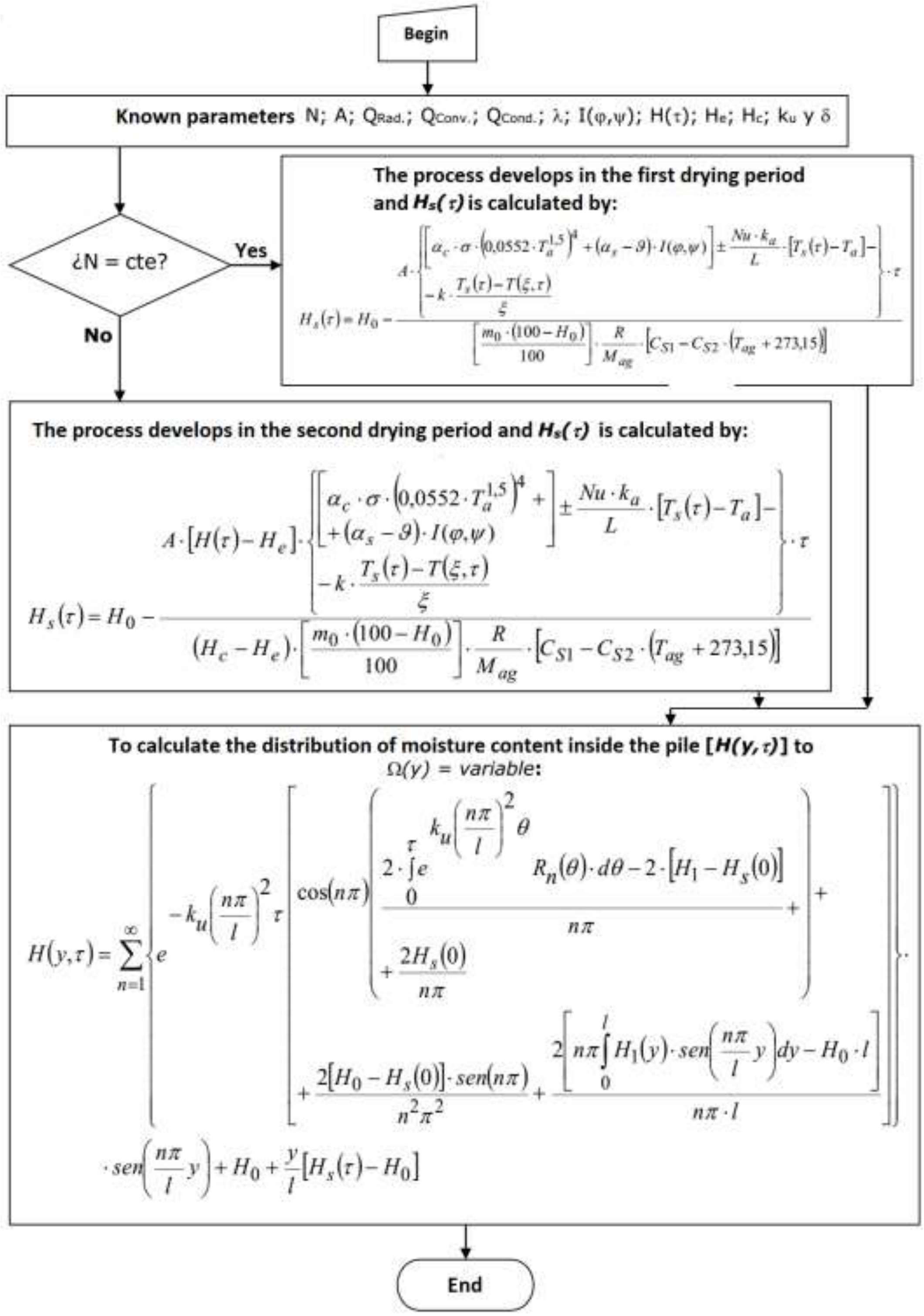

Figure 1. Systematization of the mathematical models for the natural drying of lateritic ore (Retirado-Mediaceja *et al.*, 2016).

Where:

N - Drying rate (kg/m²·s); *A* - Pile exposure area (m^2); q_{Rad} - Heat flux from radiation utilized in the natural drying of lateritic ore (W/m^2); q_{Conv} - Heat flux from convection exchanged between the air and the pile's drying surface (W/m^2); q_{Cond}: Heat transferred by conduction into the pile interior (W/m^2); λ - Latent heat of water vaporization (J/kg); $I(\varphi,\psi)$ - Global solar radiation incident on the pile's drying surface (W/m^2); $H_s(\tau)$ - Material moisture content at the pile's drying surface (at y=l) at time τ (kg/kg); H_e – material equilibrium moisture (kg/kg); H_c - Equilibrium moisture content of the material (kg/kg); k_u - Moisture conduction coefficient (m^2/s); δ - Thermal coefficient of moisture conduction (1/ºC); H_0 - Initial moisture content of the material (kg/kg); α_c - Sky absorptivity (dimensionless); σ - Stefan-Boltzmann constant ($5{,}67 \cdot 10^{-8}$ $W/m^2 \cdot K^4$); T_a - Air temperature (K); α_s - Solar absorptivity of lateritic ore (dimensionless); ϑ - Reflectivity of lateritic ore (dimensionless); *Nu* - Nusselt number (dimensionless); k_a - Thermal conductivity of air (W/m·K); *L* - Characteristic length of the drying surface (m); $T_s(\tau)$ - Temperature at the pile surface at time τ (K); *k* - Thermal conductivity of lateritic ore (W/m·K); $T(\xi,\tau)$ - Temperature inside the pile at depth ξ (K); ξ - Thickness of the material layer where heat conduction occurs (m); m_0 - Initial mass of undried material (kg); H_0 - Initial moisture content of the material (kg/kg); *R* - Gas constant (J/kmol·K); M_{ag} - Molecular weight of water (kg/kmol); C_{S1} y C_{S2} - Experimental constants (dimensionless); T_{ag} Water temperature (°C); H_e - Equilibrium moisture content of the material (kg/kg); H_c - Material moisture content at the end of the constant drying rate period (kg/kg); $H(y,\tau)$ - Moisture content inside the pile at height y and time y,τ (%).

2.2. Procedure for Model Validation

2.2.1. Experimental Conditions

The experiments were conducted using lateritic ore extracted from the active mining face. The material was transported by truck from the company mine to CEDINIQ, where it was deposited in the solar drying yard. The mineral piles were then formed using front-end loaders and other equipment recommended by Estenoz and Espinosa (2003) for this purpose.

2.2.2. Variable Selection

The drying rate of lateritic ore during the natural drying process depends on multiple variables, including: the mass of material exposed to drying, the angle of repose, the pile dimensions, and the initial and final moisture content of the material (Retirado-Mediaceja *et al*., 2017). For the experimental validation of the mathematical models, these variables were measured directly at the piles. Meteorological parameters influencing natural drying were also considered. The specifics of the variables are described below:

- *Mass of Laterite, Angle of Repose, and Pile Dimensions*

Three piles of lateritic ore were constructed: two containing 500 tonnes of material and another containing 700 tonnes. The experiments utilized a (maximal) angle of repose of 61 degrees. The general characteristics of the mineral piles are shown below (Table 1).

Table 1. Dimensions of the laterite piles subjected to natural drying

Mass (t)	Long (m)	Broad (m)	Cross Section Geometry	Orientation axis
500	140	3,20	triangular	norte-sur
700	140	5,49	triangular	norte-sur

These pile characteristics correspond to those used in the practical implementation of the natural drying process at the Cuban nickel and cobalt production company (Estenoz, 2009; Espinosa and Pérez, 2010b; Vinardell, 2011; Fuentes and Estenoz-Mejia, 2020).

- *Initial and Final Moisture Content of the Lateritic Ore*

The initial moisture content is considered an independent variable and, simultaneously, a reference parameter, as it allows for estimating the impact of natural drying on the final moisture content of the material. Its value varies randomly because it depends on the meteorological conditions of the region at the time of process implementation and the hydrogeological characteristics of the exploited deposit (Blanco and Llorente, 2004; Ochoa, 2008; De Miguel, 2007, 2009; Carmenate *et al*., 2009). Experiments were conducted with the moisture values of the lateritic ore at the time it was deposited (initial or reference values). For this purpose, three samples were taken from the longitudinal slopes of the piles. For the final moisture content, determinations were made at the same points where the initial moisture measurements were taken. The initial and final moisture values were calculated using Equations 1-4 (Martínez-Pinillos, 1997).

$$H_{bh} = \frac{m_h}{m_0} = \frac{m_0 - m_s}{m_0} \quad (1)$$

$$H_{bs} = \frac{m_h}{m_s} = \frac{m_0 - m_s}{m_s} \quad (2)$$

Where:

H_{bh} and H_{bs} – Wet basis and dry basis moisture content of the material, respectively (kg/kg); m_h Mass of water in the wet material (kg); m_0 – Initial mass of undried material (kg); m_s – Mass of dry solids in the product (kg).

The moisture contents, expressed in % (kg/kg), are related by Equations 3 and 4.

Donde:

H_{bh} y H_{bs} - humedad del material base húmeda y seca, respectivamente (kg/kg); m_h - cantidad de agua en el material húmedo (kg); m_0 - masa inicial de material sin secar (kg); m_s - masa de la materia seca en el producto (kg).

Las humedades, expresadas en *% kg/kg*, se relacionan mediante las expresiones 3 y 4.

$$H^* = \frac{100 \cdot H}{1 + H} \quad (3)$$

$$H = \frac{H^*}{100 - H^*} \quad (4)$$

Where:

H^* and H – Material moisture content (% and kg/kg).

2.2.3. Meteorological Variables

For monitoring these variables, the Davis EZ-Mount Groweather equipment belonging to the *Empresa Niquelera Comandante Ernesto Che Guevara* was used. The equipment features a data acquisition system and utilizes a sensor suite that measures and records the following meteorological variables in a computer every hour: solar radiation, cloud cover, precipitation, dew point temperature, air temperature, relative humidity, and wind direction and speed. These variables exhibit random behavior and therefore could not be preset for the experimentation. Nevertheless, their actual values were considered at the time the validation was performed.

2.2.4. Type of Experimental Design Used

Although various types of experimental designs can be employed in scientific research (Montgomery, 2004; Legrá, 2022), given the characteristics of the studied process and the available resources, a multifactorial design was implemented with the following features:

- Measurements were taken on three piles of lateritic ore to rule out the influence of the mechanical pile formation process. The piles were oriented longitudinally along a north-south axis to maximize the capture of thermal energy (solar and wind).
- Samples for measuring the moisture content of the lateritic ore were taken from the pile surfaces, thereby ensuring correct measurements with the available instrumentation.
- In each pile, three measurement points were selected at different cross-sections, and the average result was considered for subsequent analysis. This approach was taken due to the low specific and average values obtained for the coefficient of variation, which were below 5%. This confirms the quality of the measurements performed and ensures that the results obtained in one cross-section can be extrapolated to any other cross-section of the pile (Figure 2).

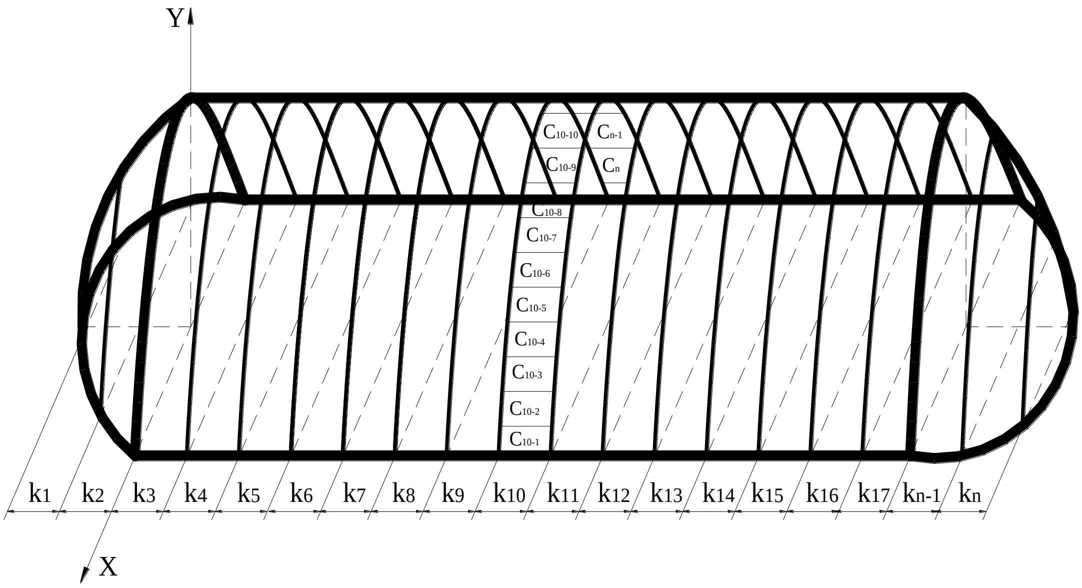

Figure 2. Representation of the cross-sections $k_1, k_2, \ldots, k_n$ and the sectors $C_1^{0-1}, C_1^{0-2}, \ldots, C_n$ formed by dividing the solar capture surface of the lateritic ore pile (Retirado-Mediaceja *et al.*, 2012a).

- The aforementioned measurements were conducted over 14 consecutive days, during which climatic variability resulted in a diverse set of experimental conditions regarding the initial material moisture content and meteorological parameters.

- Measurements were taken at equivalent points on both slopes (east and west) of the piles to verify or rule out the existence of uniformity in the material drying.
- Piles with a cross-section different from triangular or with an angle of repose different from 61 degrees were not considered for technical and economic reasons. However, this simplification does not constitute an obstacle for verifying the accuracy of the proposed theoretical models due to their physical and general nature (Retirado-Mediaceja and Legrá, 2011).

2.2.5. Experimental Design Matrix and Number of Measurements

The employed design considers the following as factors or independent variables: the distance along the "X" axis measured symmetrically from the coordinate origin (X_O y X_E), the height on the "Y" axis of the pile's drying surface (Y_S), the distance along the "Z" axis measured from the origin of the pile's lateral surface (Z_1, Z_2 y Z_3), and the time measured at 06:00 and 18:00 hours (τ_0 y τ_F).

The reference parameter is the initial moisture content of the laterite (H_0), and the dependent variable is its final moisture content (H_F). Table 2 presents the matrix of the experimental design implemented in the investigation.

Table 2. Experimental design matrix by pile

Humidity measurements to be taken in each laterite pile						
		Three samples (Z1-Z3) and the average value (P)				
Day	X (m)	Z (m)	$H_{0(Z1)}$ $H_{F(Z1)}$ (%)	$H_{0(Z2)}$ $H_{F(Z2)}$ (%)	$H_{0(Z3)}$ $H_{F(Z3)}$ (%)	$H_{0(P)}$ $H_{F(P)}$ (%)
1	X_O	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(1)}$ $H_{FP(1)}$
2	X_E	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(2)}$ $H_{FP(2)}$
3	X_O	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(3)}$ $H_{FP(3)}$
4	X_E	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(4)}$ $H_{FP(4)}$
5	X_O	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(5)}$ $H_{FP(5)}$
6	X_E	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(6)}$ $H_{FP(6)}$

7	X_O	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(7)}$ $H_{FP(7)}$
8	X_E	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(8)}$ $H_{FP(8)}$
9	X_O	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(9)}$ $H_{FP(9)}$
10	X_E	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(10)}$ $H_{FP(10)}$
11	X_O	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(11)}$ $H_{FP(11)}$
12	X_E	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(12)}$ $H_{FP(12)}$
13	X_O	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(13)}$ $H_{FP(13)}$
14	X_E	Z_{1-3}	$H_{0(Z1)}$ $H_{F(Z1)}$	$H_{0(Z2)}$ $H_{F(Z2)}$	$H_{0(Z3)}$ $H_{F(Z3)}$	$H_{0P(14)}$ $H_{FP(14)}$
Total: 84 experimental measurements						

Where:
X_O y X_E – Distance on the "X" axis measured from the origin towards the west and east slopes, respectively (m)

Z_1, Z_2, y Z_3 – Distance on the "Z" axis measured from the origin of the pile's lateral surface (m)

τ_0 y τ_F – Initial and final time measured at 06:00 and 18:00 hours of the day [h]

$H_{0(P)}$ y $H_{F(P)}$ – Average value of the initial and final moisture content of the laterite, determined experimentally (%)

2.2.6. Sampling Sufficiency and Variance Analysis

To verify the practical relevance of the established theoretical models for calculating the moisture content of lateritic ore during the natural drying process, two types of experiments can be conducted:

- Determining the material's moisture content by sampling the drying surface of the piles.
- Determining the moisture content by sampling material from both the drying surface and the interior (central part) of the piles.

However, it must be noted that when lateritic ore undergoes natural drying, it compacts and forms a nearly impenetrable crust which, according to the consulted research, makes sampling the pile interior exceedingly difficult (Espinosa and Pérez, 2010a and 2010b; Vinardell, 2011). This drawback determined that the validation of the theoretical models primarily implemented the first experiment, with the second being used to a lesser extent. In both cases, the material in the piles was homogenized during the experiments to obtain average moisture values.

Regarding the necessity of performing a variance analysis (Guerra et al., 2003), it should be emphasized that in this specific case, there is no need to infer the already known relationship between the spatial variables (x, y, z) and the temporal variable (τ) with the material moisture at each spatial point and time instant; this is evident from the H(y,τ) equation in Figure 1. On the other hand, it was also not necessary in this investigation to establish an empirical model for calculating the moisture content of the lateritic ore, for instance, using the Method of Least Squares (Legrá, 2022), because the experimental measurements performed had the sole purpose of confirming the validity of the theoretical models obtained by solving the differential equations for heat and moisture conduction with the boundary value problems posed for the investigated process.

2.2.7. Experimental Technique for Moisture Measurement

The material in the piles was turned and homogenized to obtain average moisture values. This parameter was determined by the weight difference method (Martínez-Pinillos, 1997), chosen for its reliability, simplicity, and ease of application (Miranda, 1996).

Two-kilogram samples were taken from the pile surfaces at 06:00 hours. The samples were transported in airtight containers to the laboratory, where their mass was immediately determined using a digital balance (measuring range: 0 kg to 100 kg, error: 0.058 kg). Subsequently, they were dried in a MEMMERT brand oven (temperature range: 0°C to 200°C, error: ±1%) at 105°C until the sample mass remained constant (approximately 24 hours). They were then cooled in a desiccator, the mass of the dried sample was determined, and the initial moisture content of the material was calculated.

Simultaneously, the lateritic ore piles were exposed to natural drying between 06:00 and 18:00 hours. At 18:00 hours, samples were again taken from the same sampling points, the procedure performed in the morning was repeated, and the final moisture content was determined. The effect of the natural drying process on the material's moisture content was then assessed.

3. MODEL VALIDATION

The validation of the physico-mathematical models is performed by comparing the experimentally obtained results for the material moisture content [$H_{F(P)Epx}$] with the theoretical values calculated by the models for the same experimental conditions [($H_{F(P)Teo}$]. Subsequently, the specific and average relative errors between the experimental and theoretical results are calculated, using an acceptance criterion that the average relative error is less than 10% (Viera *et al*., 1988). This specific value is chosen instead of a lower one (e.g., less than 5%) because the solution to the boundary value problem posed for the natural drying process neglects the effects of drainage and the multidimensional (2D and 3D) aspects of heat and mass transfer. These are simplifications that inherently limit the model's precision (Çengel and Ghajar, 2015).

Equations 5 and 6 proposed by Miller *et al*. (2005) are used for the error calculations. The general framework employed for the experimental validation of the models is presented in Figure 3.

$$E = \left| \frac{\left[H_{F(P)Exp.}\right] - \left[H_{F(P)Teo.}\right]}{H_{F(P)Exp.}} \right| \cdot 100 \quad (5)$$

$$E_P = \sum_{i=1}^{N_d} \left| \frac{\left[H_{F(P)Exp.}\right] - \left[H_{F(P)Teo.}\right]}{H_{F(P)Exp.}} \right| \cdot \frac{100}{N_d} \quad (6)$$

Where:

E – Specific relative error between the experimental and theoretical values of the material moisture content (%); $H_{F(P)Exp}$ – Average value of the material moisture content determined experimentally (%); E_P – Average relative error between the experimental and theoretical moisture values (%); N_d – Number of determinations (dimensionless)

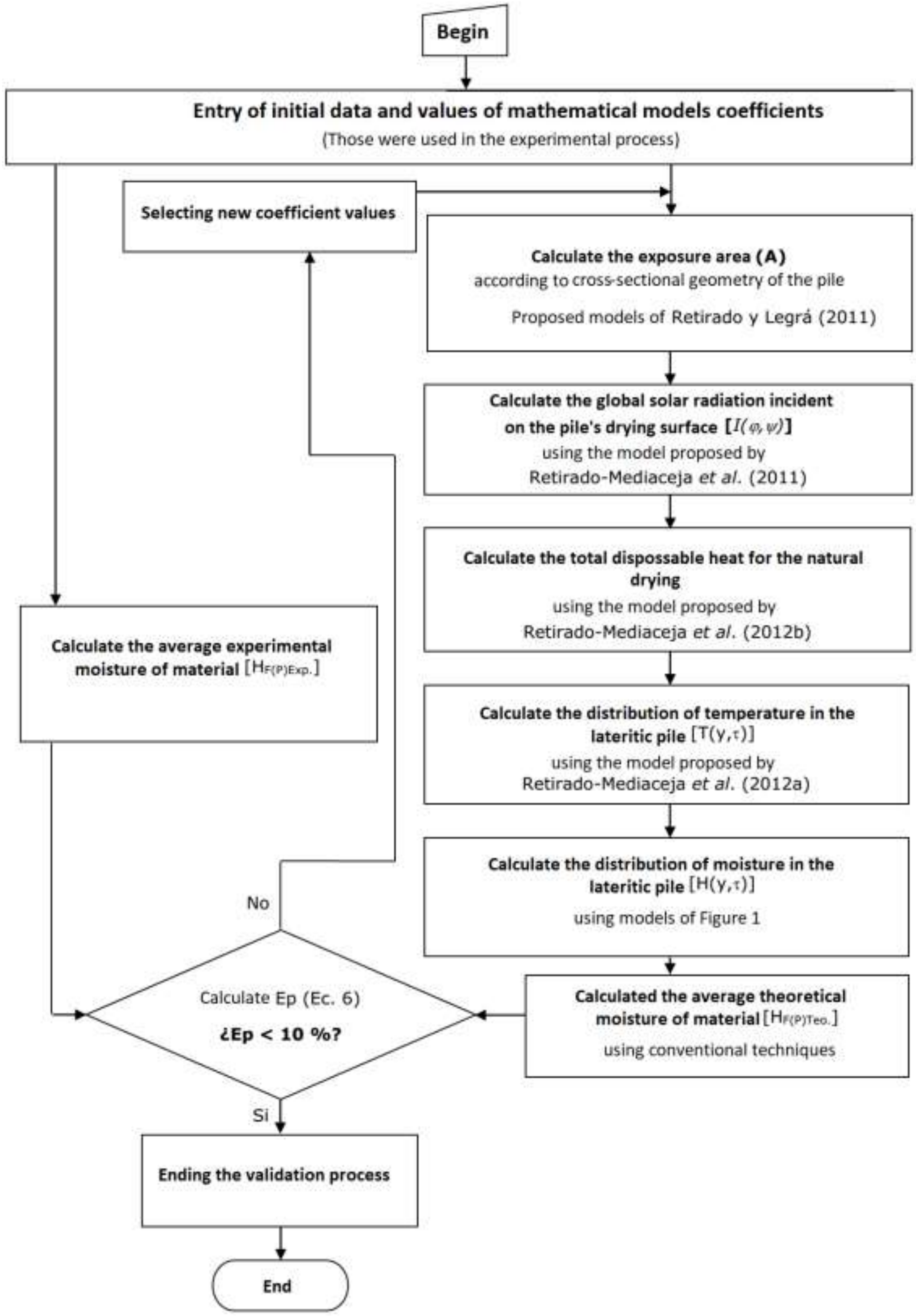

Figure 3. Framework for the validation of the physico-mathematical models.

3.2. Validation Results

Tables 3 and 5 present the moisture content values of the lateritic ore obtained experimentally during the natural drying tests and the theoretical values calculated using the mathematical models for the same experimental conditions. The experimental results [$H_{0(P)Exp.}$ y $H_{F(P)Exp.}$] are the averages for the three analyzed samples. The referenced tables show that the specific relative errors were consistently below 15%, with 73.81% of them being below 10% (Table 4). The average relative error for each individual pile is below 8%, and the overall average relative error considering all determinations is 6.57%.

Table 3. Experimental and theoretical results obtained for the moisture content

First lateritic ore pile									
Day	**$H_{0(P)\,Exp}$ (%)**	**$H_{F(P)Exp}$ (%)**	**$H_{F(P)Teo}$ (%)**	**E (%)**	**Día**	**$H_{0(P)\,Exp}$ (%)**	**$H_{F(P)Exp}$ (%)**	**$H_{F(P)Teo}$ (%)**	**E (%)**
1	34,05	27,81	29,61	6,48	8	38,04	31,06	34,06	9,66
2	32,07	29,46	27,40	6,99	9	31,42	26,21	26,66	1,72
3	31,48	27,02	26,74	1,04	10	36,93	30,62	32,82	7,18
4	34,81	30,29	30,46	0,56	11	31,36	25,15	26,60	5,77
5	31,72	27,90	27,00	3,23	12	27,73	22,61	22,55	0,27
6	35,13	30,81	30,81	0,00	13	25,63	19,61	20,20	3,01
7	32,45	30,68	27,82	9,32	14	35,31	29,51	31,02	5,12
Average relative error between experimental and theoretical values (E_p = 4,31 %)									
Second lateritic ore pile									
Day	**$H_{0(P)\,Exp}$ (%)**	**$H_{F(P)Exp}$ (%)**	**$H_{F(P)Teo}$ (%)**	**E (%)**	**Día**	**$H_{0(P)\,Exp}$ (%)**	**$H_{F(P)Exp}$ (%)**	**$H_{F(P)Teo}$ (%)**	**E (%)**
1	31,88	30,66	27,18	11,35	8	30,62	27,72	25,77	7,03
2	29,99	29,05	25,06	13,73	9	28,09	24,24	22,94	5,36
3	35,30	27,43	31,01	13,05	10	37,39	31,54	33,34	5,71
4	29,63	27,61	24,66	10,68	11	27,83	24,74	22,65	8,45
5	31,16	26,40	26,37	0,11	12	36,73	33,01	32,60	1,24
6	31,11	30,39	26,32	13,39	13	23,89	18,21	18,29	0,44
7	31,73	25,04	27,01	7,870	14	33,61	26,77	29,11	8,74
Average relative error between experimental and theoretical values (E_p = 7,65 %)									
Design of experiments factor values for stacks 1 and 2									

X_O = +0,8 m; X_E = -0,8 m; Z_1 = 35 m; Z_2 = 70 m; Z_3 = 105 m; Y_s = 1,443 m; τ_0 y τ_F = 0 y 12 h									
Third lateritic ore pile									
Day	**$H_{0(P)\,Exp}$ (%)**	**$H_{F(P)Exp}$ (%)**	**$H_{F(P)Teo}$ (%)**	**E (%)**	**Día**	**$H_{0(P)\,Exp}$ (%)**	**$H_{F(P)Exp}$ (%)**	**$H_{F(P)Teo}$ (%)**	**E (%)**
1	36,32	30,61	33,88	10,68	8	33,05	27,50	30,36	10,40
2	36,61	33,89	34,19	0,89	9	33,96	27,42	31,34	14,30
3	35,50	32,11	33,01	2,80	10	34,94	30,85	32,40	5,02
4	42,77	36,56	40,77	11,52	11	32,22	27,75	29,47	6,20
5	39,80	34,68	37,61	8,45	12	22,74	18,38	19,18	4,35
6	34,23	30,54	31,63	3,57	13	29,02	22,67	26,01	14,73
7	36,99	33,93	34,60	1,97	14	25,07	19,13	21,71	13,49
Average relative error between experimental and theoretical values (E_p = 7,74 %)									
Average relative error between experimental and theoretical values X_O = +1,372 m; X_E = -1,372 m; Z_1 = 35 m; Z_2 = 70 m; Z_3 = 105 m; Y_s = 2,476 m; τ_0 y τ_F = 12 h									

These values indicate a satisfactory correspondence between the moisture content results obtained experimentally during natural drying and the theoretical values calculated with the established models. The specific relative errors calculated for each of the moisture levels listed in Table 3 follow the distribution shown in Table 4.

Table 4. Distribution of the specific relative errors

Laterite pile 1		**Laterite pile 2**		**Laterite pile 3**	
Interval (%)	**PRE (%)**	**Interval (%)**	**PRE (%)**	**Interval (%)**	**PRE (%)**
0 ≤ E ≤ 5	50	0 ≤ E ≤ 5	21,43	0 ≤ E ≤ 5	35,71
5 < E ≤ 10	50	5 < E ≤ 10	42,86	5 < E ≤ 10	21,43
10< E ≤ 15	0,0	10 < E ≤ 15	35,71	10 < E ≤ 15	42,86
Distribution of the point relative error for the three stacks together					
0 ≤ E ≤ 5 (PRE = 35,71 %)		5 < E ≤ 10 (PRE = 38,10 %)		10< E ≤ 15 (PRE = 26,19 %)	

Where:
PRE – Percentage represented by the errors within the considered interval (%).

Considering the overall agreement of 93.43% (100% - 6.57%) achieved with the established models for calculating the material moisture content, the distribution of the calculated specific relative errors, and their low average values (Tables 3-5), it can be affirmed that the physico-mathematical models established for the energy analysis of the natural drying process of lateritic ore have an acceptable accuracy for thermal engineering calculations (greater than 90%) and are, therefore, valid for the purposes for which they were created.

In this regard, literature addressing the mathematical modeling of industrial processes specifies that for most engineering calculations (except in processes and installations that, due to their intrinsic characteristics, require high calculation precision, e.g., nuclear reactors), an approximation of 90% is considered satisfactory, because the results are always influenced by errors inherent to the experimentation process (Tikhonov, 1978; Lyashko, 1984).

Table 5. Experimental and theoretical results obtained for the moisture content of the lateritic ore at different depths

Depth at which the moisture content of the material was measured (m)	East slope of the mineral pile			
	6:00 h	18:00 h		
	$H_{0(P)\ Exp}$ (%)	$H_{F(P)Exp}$ (%)	$H_{F(P)Teo}$ (%)	E (%)
Surface (0,0)	32,46	27,59	28,44	3,08
(-0,3)	32,46	28,15	28,93	2,77
(-0,6)	32,46	28,37	29,42	3,70
(-0,9)	32,46	29,22	29,91	2,36
(-1,2)	32,46	29,43	30,41	3,33
(-1,5)	32,46	32,18	30,90	3,98
Depth at which the moisture content of the material was measured (m)	**West slope of the mineral pile**			
	6:00 h	**18:00 h**		
	$H_{0(P)\ Exp}$ (%)	$H_{F(P)Exp}$ (%)	$H_{F(P)Teo}$ (%)	E (%)
Surface (0,0)	32,46	26,09	26,25	0,61
(-0,3)	32,46	26,61	27,01	1,50
(-0,6)	32,46	27,17	27,76	2,17

	(-0,9)	32,46	28,19	28,52	1,17
	(-1,2)	32,46	30,04	29,28	2,53
	(-1,5)	32,46	30,91	30,04	2,81
Average relative error between experimental and theoretical values of laterite moisture (E_p = 2,50 %)					

Note: Depth was measured from the surface to the base of the pile.

Finally, it is commendable to highlight that, according to the results from both experiments—on the surface and within the piles (Tables 3 and 5)—the models respond satisfactorily to the physical reality of the process. However, it is acknowledged that their scope is limited, as they are one-dimensional (1D) models attempting to explain a multidimensional physical phenomenon of heat and mass transport (drying in 2D and 3D) taking place within a control volume (the mineral pile). Therefore, future research will be directed towards developing and validating a two-dimensional model of the process under study.

4. CONCLUSIONS

- The implementation of a multifactorial experimental design enables the experimental validation of the established physico-mathematical models for the energy analysis of the natural drying process of lateritic ore. The results demonstrate a satisfactory correspondence between the experimentally determined material moisture values and the theoretical values calculated by the models under similar implementation conditions.

- The experimental validation corroborates that the natural drying models for lateritic ore describe the physical reality of the process with high accuracy (93.43%). This accuracy can be improved through multidimensional (2D and 3D) modeling of the heat and mass transfer processes that characterize the subject of study. This relevant aspect will be the focus of analysis in future research.

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Información adicional

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YRM: Realiza la revisión bibliográfica; diseña y escribe el artículo propuesto. Participó en la formalización de los modelos, el análisis de los resultados, redacción del borrador del artículo, la revisión crítica de su contenido y en la aprobación de la versión final del manuscrito. AALL: concepción general y ejecución de la investigación, análisis matemático del objeto de estudio y formalización de los modelos, escritura y revisión crítica, aprobación de la versión final del manuscrito. YCM: análisis de los resultados, escritura, revisión crítica y aprobación final del manuscrito. HLLA: búsqueda bibliografía, análisis de los resultados, revisión crítica y aprobación final del manuscrito. MFSC: búsqueda bibliografía, análisis de los resultados y revisión del informe final. WGQS: búsqueda bibliografía, análisis de los resultados y revisión del informe final.

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