# The auto-regressive modeling approach applied to denoise moroccan resistivity data phosphate disturbances map

Saad BAKKALI[1] saad.bakkali@menara.ma
Mahacine AMRANI[2] amrani.mahacine@menara.ma

**Abstract**
Several methods are currently used to optimize edges and contours of geophysical data maps. A resistivity map was expected to allow the electrical resistivity signal to be imaged in 2D in Moroccan resistivity survey in the mining domain. A Schlumberger resistivity survey over an area of 50 hectares was carried out to detect resistivity anomalies associated to phosphate disturbances. To filter out white noise from the resistivity data was used a method based on the auto-regressive model (ARM) approach. It was implemented by means of AutoSignal routine. The auto-regressive filtering output of the phosphate deposit disturbances map obtained by this approach shown that ARM gives an effective filtering method which attenuates considerably the noise represented by minor dispersed and random disturbances; it gives a significant reduction of distortion to the shape of the original resistivity signal.

**Key words**
Resistivity, Schlumberger, phosphate, auto-regressive, model, filtering, Morocco.

Recibido: diciembre 2007 / Aceptado: enero 2008

[1] Geosciences & Environment Group. Faculty of Sciences & Techniques, Tangier, Morocco.
[2] Mahacine Amrani. Engineering Process Department. Faculty of Sciences & Techniques, Tangier, Morocco

 ISSN 1993 8012

# Modelo auto-regresivo aplicado a la eliminación de ruidos de los mapas de anomalías de resistividad en fosfatos asociadas a bolsones estériles


**Resumen**

Actualmente se emplean varios métodos para perfeccionar los bordes y contornos de mapas geofísicos. En Marruecos se realizó una prospección de fosfatos empleando el método eléctrico de resistividad; para ello se utilizó un dispositivo Schlumberger, cubriendo una superficie de 50 hectáreas, con el objetivo de detectar anomalías asociadas a bolsones estériles en los depósitos de fosfatos. Para la eliminación del ruido blanco se empleó un método basado en un modelo auto-regresivo (ARM, siglas en inglés); este se implementó por medio del algoritmo AutoSignal; el resultado muestra que el método ARM es efectivo para el filtrado, atenúa considerablemente el ruido producido por bolsones dispersos y de pequeño tamaño, además brinda una reducción considerable de la distorsión de la forma de la señal de resistividad original.



**Palabras clave**

Resistividad, Schlumberger, fosfato, auto-regresivo, modelo, filtrado, Marruecos.


Received: December 2007 / Accepted: January 2008

## INTRODUCTION

The study area is the Oulad Abdoun phosphate basin which contains the Sidi Chennane deposit. The Sidi Chennane deposit is sedimentary and contains several distinct phosphate-bearing layers. These layers are found in contact with alternating layers of calcareous and argillaceous hardpan. However, a new deposit contains many inclusions or lenses of extremely tough hardpan locally known as *derangements* or disturbances (Figure 1), found throughout the phosphate-bearing sequence. The hardpan pockets are normally detected only at the time of drilling (Kchikach et al., 2002).


Figure 1. Example of disturbance affecting the phosphate strata

Direct exploration methods such as well logging or surface geology are not particularly effective to estimate phosphate reserves. They interfere with field operations and introduce a severe bias in phosphate reserves estimates (Figure 2).

Selection of the study area was based on its representativeness and the apparent resistivity profiles were designed to contain both disturbed and enriched areas (Figure 3). The sections were also calibrated by using vertical electrical soundings (Figure 4) (Bakkali et al., 2006a). Schlumberger array of AB=120 m was chosen for this study. The required restrictions were made to meet the recommendations of our business associates: OCP Group (Office Cherifien des Phospahtes) which is the Moroccan company specialized in mining, processing and marketing phosphates.

High values of apparent resistivity were encountered due to the presence of near-vertical faulting between areas of contrasting resistivity, and fault zones which may contain more or less highly conducting fault gouge. The gouge may contain gravel pockets or alluvial material in a clay matrix. Such anomalous sections are also classified as disturbances. Apparent resistivity values in these local profiles exceeded 200 $\tilde{\Omega}.m$. The apparent resistivity map (Figure 5) obtained from the survey is actually a map of discrete potentials on the free surface, and any major singularity in the apparent resistivities due to the presence of a perturbation will be due to the crossing from a normal into a perturbed area or vice versa. Interpretation of resistivity anomalies is the process of extracting information on the position and composition of a target mineral body in the ground (Bakkali, 2006). In this particular case the targets were essentially the inclusions called perturbations. The amplitude of an anomaly may be assumed to be proportional to the volume of a target body and to the resistivity contrast with the mother lode. If the body has the same resistivity as the mother lode no anomaly will be detected. It assumed in the first approach that the resistivity anomalies would be representative of the local density difference between the disturbances and the mother lode. Level disturbance of the anomalous zones is proportional to resistivity intensity (Figure 6).

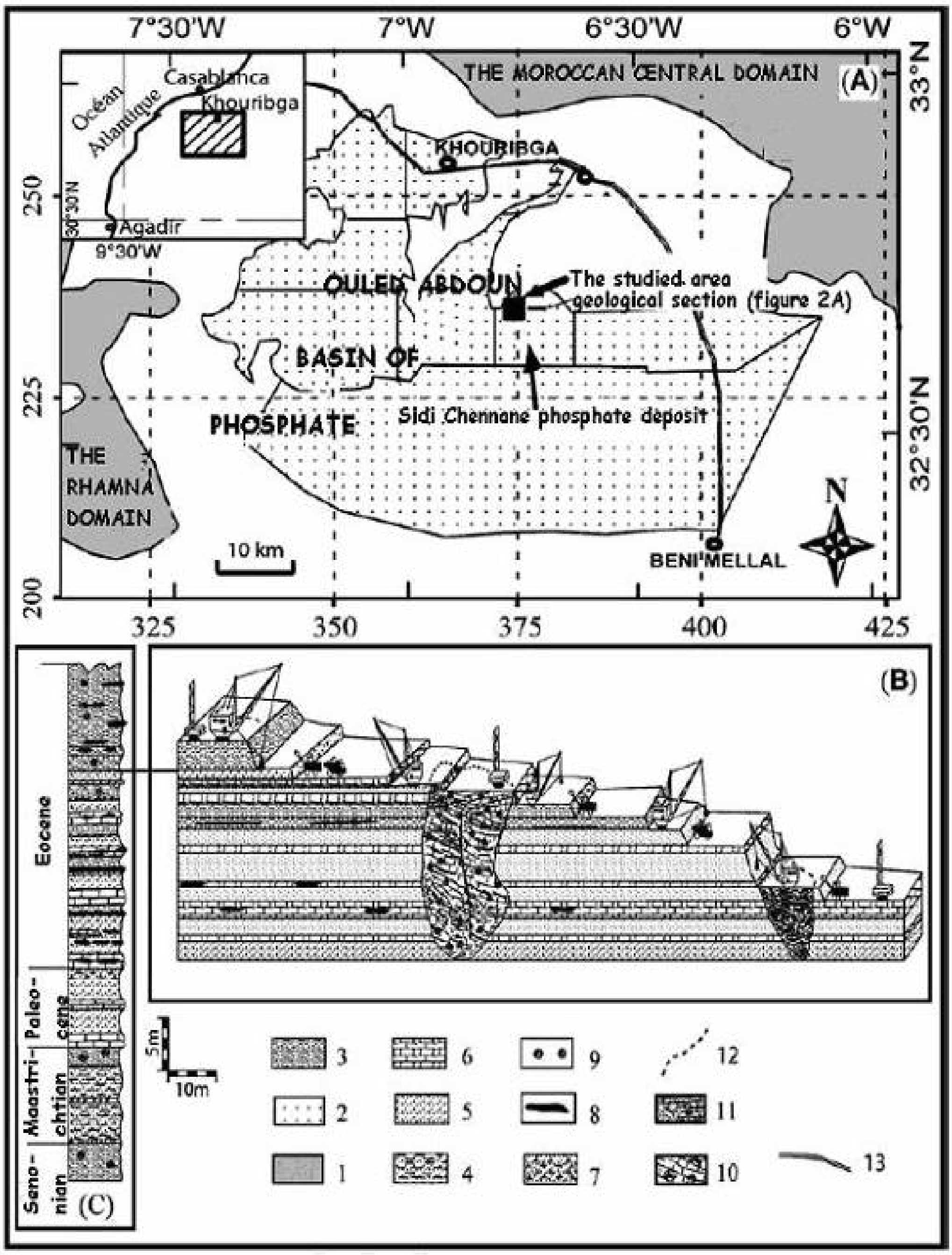


Figure 2. (A) Location of the studied area in the sedimentary basin of Ouled Abdoun. (B) Section showing the disruption of the exploitation caused by disturbances. (C) Stratigraphical log of the phosphatic series of Sidi Chennane: (1) Hercynian massif; (2) Phosphatic areas; (3) Marls; (4) Phosphatic; Marls; (5) Phosphatic layer; (6) Limestones; (7) Phosphatic limestone; (8) Discontinuous silex bed; (9) Silex nodule; (10) Disturbance formed exclusively of silicified limestone; (11) Disturbance: comprise by a blend of limestone blocks, marls and clays; (12) Disturbance limit; (13) Roads.

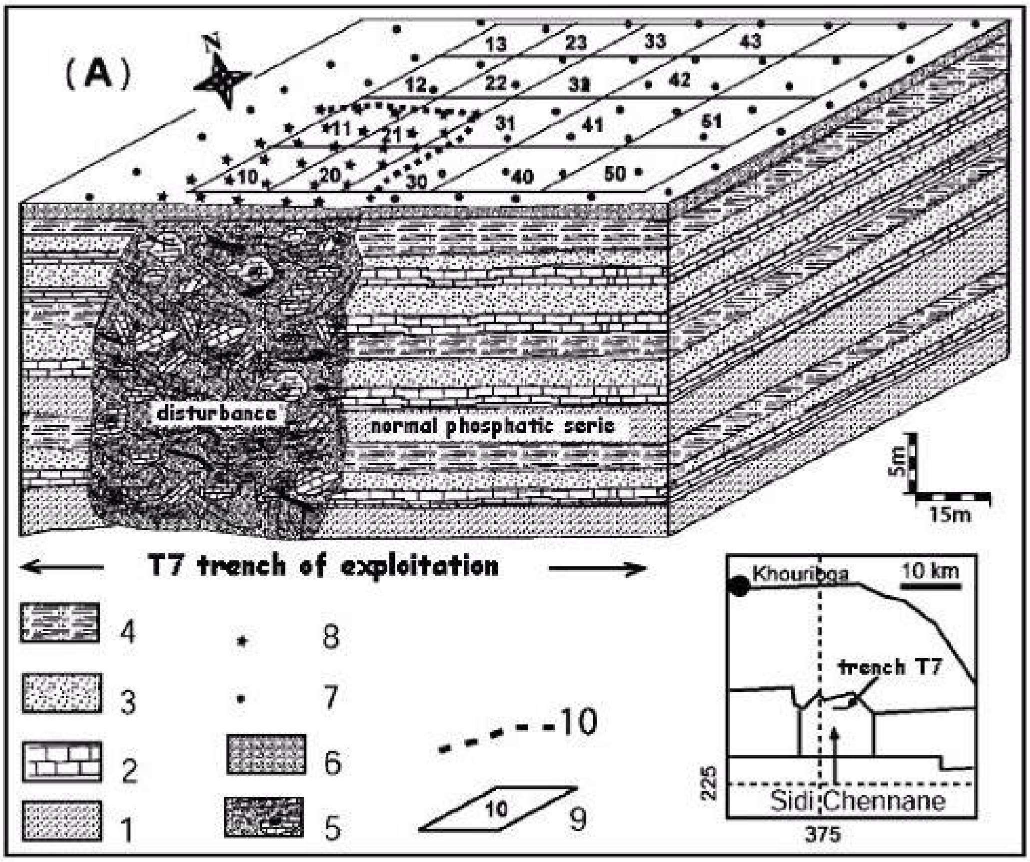


Figure 3. (A) Geological section of the T7 exploitation trench showing a disturbance and arrangement of the soundings tests. Apparent resistivity profiles positions from the deranged zone to a normal phosphatic series: (1) phosphatic marls; (2) limestones; (3) phosphatic layer; (4) marls; (5) disturbance; (6) Quaternary cover; (7) borehole crossing a normal phosphatic series; (8) borehole crossing a disturbance; (9) measures loop number 10; (10) disturbance limit.

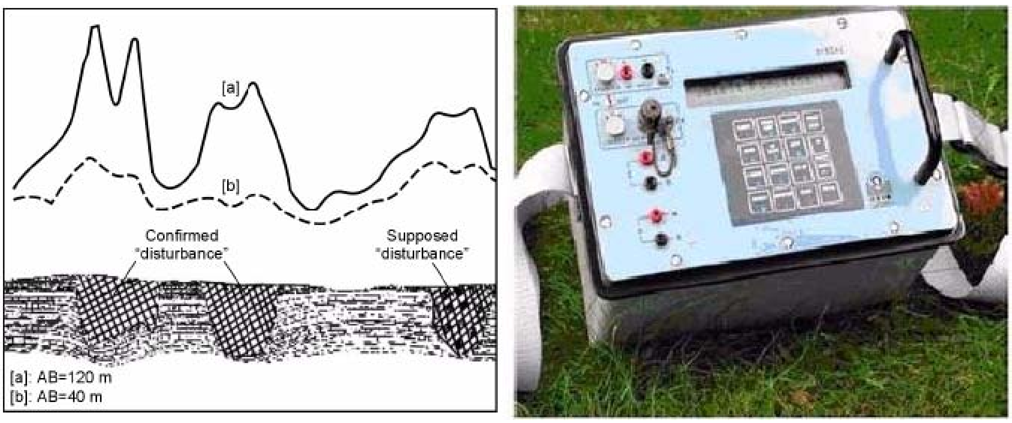


Figure 4. A synthetic apparent resistivity traverses over three disturbances (left side) and the Syscal resistivity meter used in the study (right side).

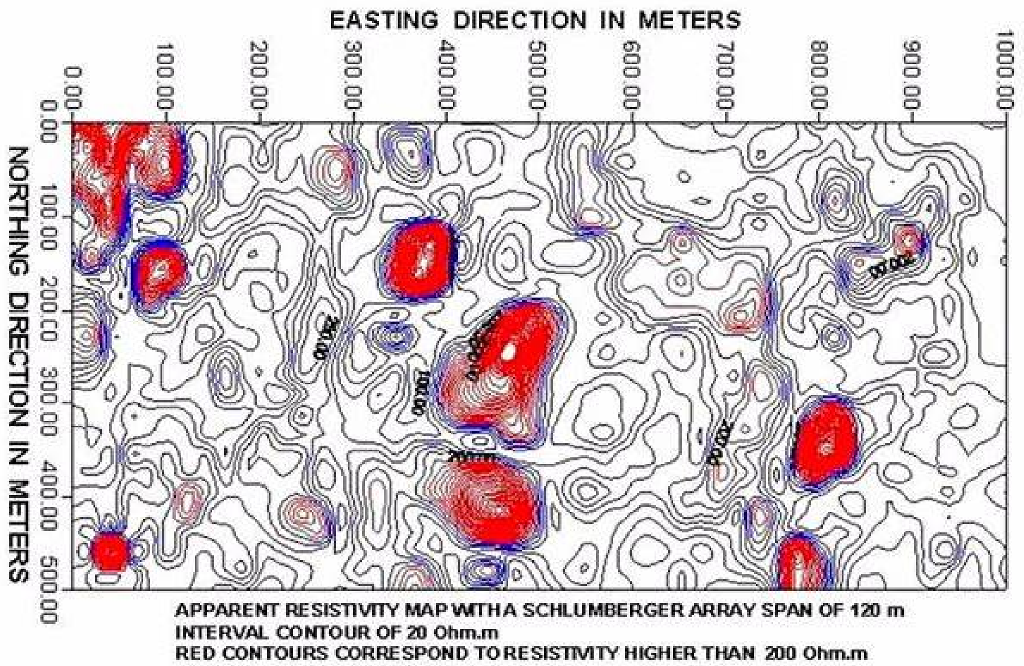


Figure 5. A map of resistivity anomalies for AB=120 m.

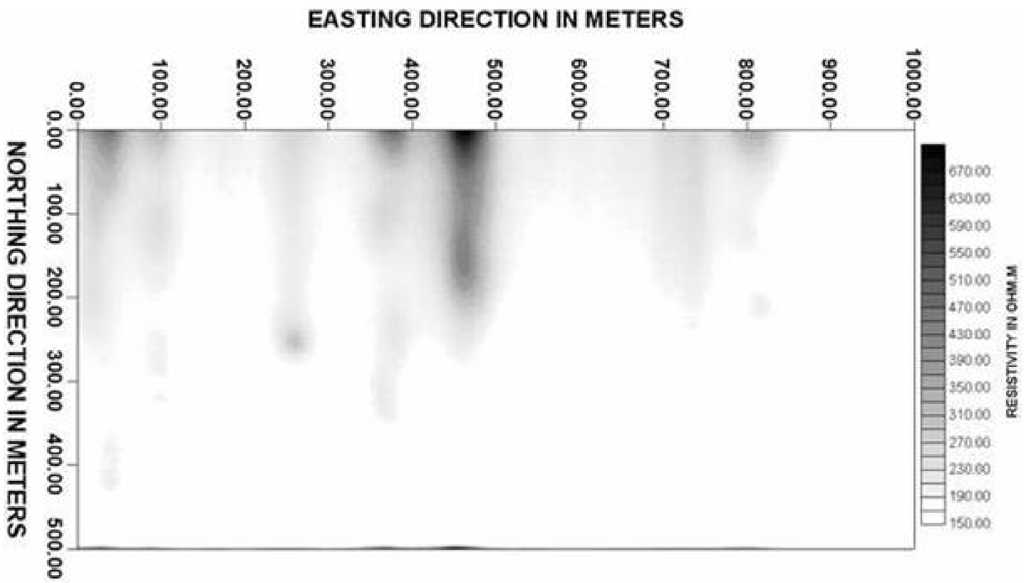


Figure 6. A map of the disturbed noisy phosphate zones corresponding to Figure 5.

Resistivity data collected in the survey are often contaminated with white noise and artifacts coming from various sources. The presence of white noise in data resistivity distorts the characteristics of the geophysical signal, resulting in poor quality of any subsequent processing. Consequently, the first step in processing geophysical data is removal of noise, maintaining signal sharp variations.

The ARM filtering approach has become a powerful signal and image processing tool with many applications in several scientific areas. This research will include the analysis of resistivity data map by using the ARM filtering approach to denoise anomalous zones map of phosphate deposit disturbances.

## THE AUTO-REGRESSIVE MODEL APPROACH

The spatially ARM assumes a spatially correlated response also dependent on attributes of the neighboring points (Ripley, 1988). It is also known in the filter design techniques as an infinite impulse response filter or an all pole filter, and is sometimes known as a maximum entropy model in geophysics applications. The name auto-regressive comes from the fact that $\vec{\rho}$ is regressed on its previous values. In analysis of spatial geophysical data, numerous attempts are made to explicitly include a spatial component in the prediction models. Modeling consists of two steps, in models with spatially correlated residuals and with auto-regressive disturbance (Bowman et al., 1997). First, the response variable is treated as non-spatial and a linear model is applied. Then, the residuals of a linear model on training data are assumed spatially correlated and their dependence is modeled through a matrix of weights $W$ using an auto-regressive approach. The ARM represents in fact one type of Markovian field (Berchtold, 1996 ; Congalton, 1988 ; Zhang et al., 2002 ) and is defined as

$$\vec{\rho} = \nu W \vec{\rho} + \varepsilon \qquad (1)$$

where $\vec{\rho}$ is a vector of apparent resistivity data grid values in the random field (representing Schlumberger apparent resistivity data traverse), $\nu$ is a parameter of spatial autocorrelation (0-0.25 in square grid). $W$ is a matrix of weights (1 for rook's case neighbours and zeros otherwise) also identified as the matrix of the auto-regression coefficients. $\varepsilon$ is a vector of Gaussian random deviations which is also assumed

 ISSN 1993 8012

to be Gaussian white noise. This noise could be considered as a constant, affecting a wide range of scales. Such contamination can be caused by a sensor guard and a float used to prevent mechanical damage to the profiler during the measurements, but actual sources of possible white noise have not been identified yet. Noise may be have been introduced by the electronics, the environment or also by operators. The solution to Equation 1 requires the inversion of $(I - \nu W)$ ($I$ is the unit matrix), which becomes practical even with simulation of small areas (Equation 2).

$$\vec{\rho} = (I - \nu W)^{-1} \varepsilon \qquad (2)$$

The problem in the spatially ARM analysis is to derive the best values for the matrix of the auto-regression coefficients given a vector $\vec{\rho}$ (Kelly et al., 1998; Cleveland et al., 1988). However, algorithms with reduced computational complexity for solving $\vec{\rho}$ with adequate numerical accuracy do exist and have been applied in generating realisations of Gaussian random processes (Billings et al., 2002).

In others terms mathematical auto-regressive resistivity data modeling assumes that each value of the series (corresponding to the Schlumberger resistivity traverses survey) depends only on a weighted sum of the previous values of the same series plus noise. If $\vec{\rho}(k)$ is the $k^{th}$ value of the series, the auto-regressive model of order $N$ is given by:

$$\vec{\rho}(k) = \sum_{p=1}^{N} \nu_p \vec{\rho}(k - p) + \varepsilon(k) \qquad (3)$$

The spatial auto-regressive method is considered suitable, especially for cases where the shape of the semivariogram is not explicitly known, even though the relation between the Bessel semi-variogram model and parameter $\nu$ has been established (Griffith, 2000).

 ISSN 1993 8012

## METHODOLOGY AND PROCEDURE

The resistivity data base is a compilation of 51 lines at a spacing of 20 m. There were 101 stations at 5 m distance for every traverse, which makes 5 151 stations all together in the resistivity survey. The auto-regression coefficients were computed from least-squares matrix procedures. The least-squares methods yield results that are a function of how resistivity data are treated at the bounds (matrix size) as well as whether the resistivity data matrix or normal equations are fitted. Least-squares methods offer in-situ separation of signal and noise through singular value decomposition. The auto-regressive model approach problem is reduced to finding the optimal auto-regression coefficients matrix $W$ over the complete resistivity data. We calculated the output auto-regressive model filtered signal using AutoSignal routine for each resistivity traverse (Figure 7) (Systat, 2002). Then we deferred all the results to build a regular map which represent the auto-regressive model filtering and denoising map of the phosphate deposit disturbances (Figure 8). The advantage of the auto-regressive model filter is the ability to preserve higher moments in the resistivity data and thus reduce smoothing on peak heights. It is specifically suitable for denoising, filtering and analyzing problems and potential singularities (Isaaks et al., 1988).

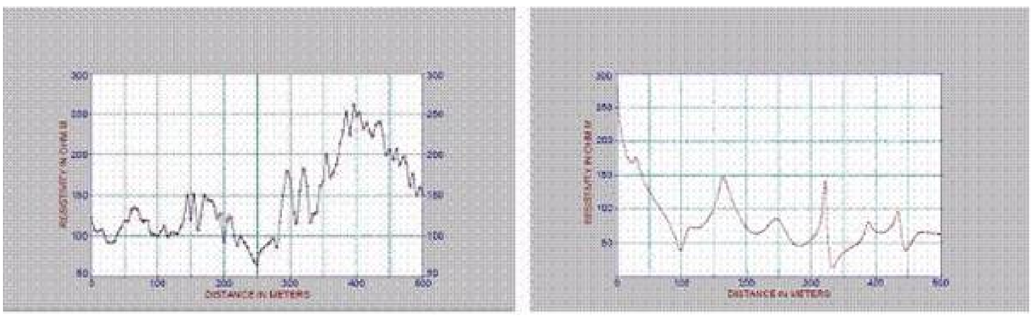

Figure 7. Example of real resistivity traverse data of the survey (left side) and the corresponding auto-regressive modeling output filtering (right side).

## RESULTS AND CONCLUSIONS

Figure 8 gives an indicator of the variation level of density differences between the disturbances and the normal phosphate-bearing rock. This enables us to identify the anomalies area which turned out to be strongly correlated with the disturbances. It represents an effective noise filtered indicator of the intensity level of disturbance.

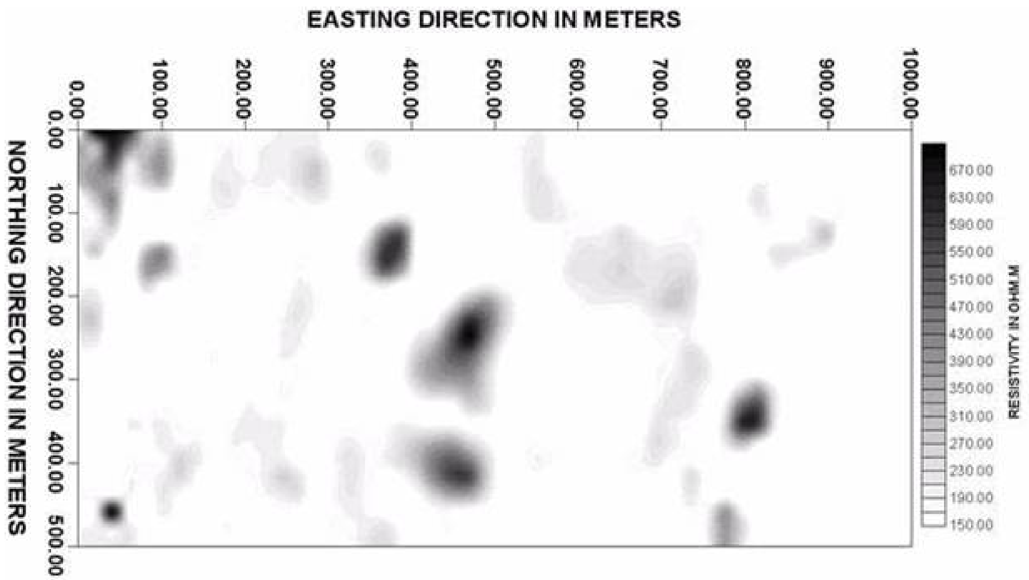


Figure 8. The auto-regressive filtering output of the phosphate deposit disturbances map given in Figure 6.

The ARM represents an effective filtering method which makes possible to attenuate considerably the noise represented by minor dispersed and random disturbances, comparatively to others methods used and tested previously (Bakkali et al., 2006b and Bakkali, 2007). Scanning and denoising the anomalous bodies is the overall effect. Compared to classical approaches used in filtering and denoising geophysical data maps, the advantage of the ARM filtering method lies on significant reduction of distortion to the shape of the original resistivity signal.

The ARM filtering output of the apparent resistivity map corresponding to the ARM filtering output of the anomalous phoshate deposit map obtained from this technical tool represents the crossing dominate area from a normal into a perturbed area or vice versa. Moreover, the level of real physical disturbance is very clearly shown. The proposed filtering and denoising method using ARM filtering tends to give a real estimation of the surface of the phosphate deposit disturbances zones with a significant reduction of noise. The level disturbance resulting from this method is also more defined in all the disturbed zones.

**REFERENCES**


BAKKALI, S. 2006. Optimización de anomalías en un deposito de fosfatos de Marruecos mediante el método del sombreado. *Studia Geologica Salmanticensia*, 42 :21-32.

---------- 2007. Enhancement of edges of Sidi Chennane phosphate disturbance using sunshading responses of resistivity data. *Russian Geology & Geophysics, 48 :775-781.*

BAKKALI, S. & L. BAHI, 2006 a. Enhancement edges of Sidi Chennane phosphate deposit disturbance using non-parametric regression analysis of resistivity data, Annals of the Faculty of Engineering Hunedoara, Tome IV Fasc. 3, 135-140.

---------- 2006 b. Cartographie des «dérangements » de séries phosphatées par mesures de résistivités électriques. *Journal des Sciences Pour l'Ingénieur*, 6 :1-10.

BERCHTOLD, A., 1996. Modélisation autorégressive des chaînes de Markov: Utilisation d'une matrice différente pour chaque retard. *Revue de Statistique Appliquée*, XLIV (3) :5-25.

BILLINGS, S.D; BEATSON, R.K. & NEWMAN G.N., 2002 a. Smooth fitting of geophysical data using continuous global surfaces. *Geophysics*, 67 :1823-1834.

---------- 2002 b. Interpolation of geophysical data by continous global surfaces. *Geophysics*, 67 :1810-1822.

BOWMAN, A.W. & AZZALINI, A. 1997. Applied Smoothing Techniques for Data Analysis. *Oxford Statistical Science Series*, 18. Clarendon Press, Oxford, UK.

CLEVELAND, W.S & DEVLIN, S.J. 1988. Locally weighted regression: an approach to regression analysis by local fitting. *Journal of the American Statistical Association,* 83 :596-610

CONGALTON, R.G., 1988. Using spatial autocorrelation analysis to explore the errors in maps generated from remotely sensed data. *Photogrammetric Engineering and Remote Sensing*, 54 (5) :587-592

GRIFFITH, D.A., 2000. A linear regression solution to the spatial utocorrelation problem. *J.Geograph Syst*, 2 :141-156.

ISAAKS, E.H. & SRIVASTAVA, R.M. 1988. Spatial continuity for probabilistic and deterministic geostatistics. *Mathematical Geology*, 20 :313-341.

KCHIKACH, A., JAFFAL, M., AIFA, T. & BAHI, L. 2002. Cartographie de corps stériles sous couverture quaternaire par méthode de résistivités électriques dans le gisement phosphaté de Sidi Chennane (Maroc). Comptes Rendus. *Geosciences*, 334 :379-386.

KELLY P.; BARRY, R.; CLAPP, J. & RODRIQUEZ, M. 1998. Spatiotemporal auto-regressive models of neighborhood effects. *Journal of Real Estate Economics, 17* :15-33

RIPLEY, B. D., 1988. Statistical Inference for Spatial Processes. Cambridge University Press, 148 p.

SYSTAT 2002. About AutoSignal V1.6 software, Copyright 2002 AISN Software Inc.

ZHANG, J. & M. F. GOODCHILD. 2002. *Uncertainty in geographical information*, Taylor & Francis, London-New York, 266 p.